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NOLTR 62-36

SCALING THE ENERGY SPECTRA OF UNDERWATER EXPLOSION SHOCK WAVES

NOL

2 APRIL 1963

UNITED STATES NAVAL ORDNANCE LABORATORY, WHITE OAK, MARYLAND

NOLTR 62-36

SCALING THE ENERGY SPECTRA OF UNDERWATER EXPLOSION SHOCK WAVES

By

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ABSTRACT: Scaling factors with which the energy spectrum for an underwater shock wave can be scaled from one set of conditions (i.e., explosive composition, charge weight, and range) to a second set of conditions are derived. Analogous scaling factors for an ideal acoustic wave are also shown. The spectrum measured at 1 yard from a 1-lb charge is scaled to two other conditions (viz., 100 yards from a 1-lb charge, and 280 yards from a 1.8-lb charge) for which measured values are also available. In both cases the explosion scaling is in agreement with the data whereas the ideal acoustic scaling gives spectrum levels that are too high at the higher frequencies.

The allowable ranges of variables for the spectrum scaling functions shown here are estimated to be:

$0.5 <$ frequency (kc) < 20

$4 \times 10^{-4} \leq W^{1/3}/R \leq 3$, where W is charge weight in pounds and R is range in yards.

Details of the spectral analysis of measured shock waves and a method of estimating the effect of explosive composition on spectrum level are given in appendices.

PUBLISHED JUNE 1963

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SCALING THE ENERGY SPECTRA OF UNDERWATER EXPLOSION SHOCK WAVES

In the investigation of explosive charge designs for explosive echo ranging (EER) under Task No. RUME-4-E-000/212-1/F008-10-04, Problem Assignment 002, it is often desirable to compare the shock wave energy spectra for different sets of experimental variables or to scale spectra from one set of conditions to another. In this report generalized scaling factors for exponential explosion pulses are shown, and the differences between the scaling laws for explosion shock wave and ideal acoustic pressure waves are discussed.

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LIST OF SYMBOLS

c - sound velocity in water

f - cyclical frequency, cyles per second

k - coefficient in similitude equation for shock wave maximum pressure

m - coefficient in similitude equation for shock wave time constant

p - maximum pressure of explosion shock wave

R - range from explosion

W - explosive charge weight, lb

$E(f)$ - energy spectrum function for exponential wave

K_E - energy scaling factor (spectrum scaling)

K_f - frequency scaling factor (spectrum scaling)

C, X_E, X_f - spectrum scaling factors relating explosive compositions, decibels (defined in Eqs. A.2-A.4)

α - exponent in similitude equation for shock wave maximum pressure

β - exponent in similitude equation for shock wave time constant

ϵ - difference (in decibels) between spectra for two explosive materials, $= C + X_E + X_f$

η - spectrum scaling function (defined in Eq. A.5)

θ - decay constant of exponential shock wave

ρ - density of water

ω - angular frequency, radians, $= 2\pi f$

SCALING THE ENERGY SPECTRA OF UNDERWATER EXPLOSION SHOCK WAVES*

1. INTRODUCTION

1.1 At very short ranges from an underwater explosion the relationships between charge weight, range, and explosive composition have been well established from numerous measurements of the damage potential of a charge fired underwater (ref. (1))**. A number of measurements have also been made at great distances from small charges used as acoustic sources (e.g., refs. (2), (3), and (9)), and it has been found that at ranges of many miles acoustic wave propagation laws adequately describe the shock wave propagation. At intermediate ranges, however, within a few miles of a charge weighing several pounds, the relationships between explosion and acoustic waves have not been well defined. Theory predicts that the shock wave propagation laws will approach asymptotically the ideal acoustic laws, but Arons (ref. (4)) found that the near-field similitude laws described the explosion wave form out to greater ranges than expected. It is this intermediate region, which is "short-range" from the point of view of most previous explosion acoustics studies and is "long-range" from the point of view of explosion damage studies, that is of primary interest for explosives echo ranging application.

1.2 Within this intermediate region acoustic approximations are suitable for some of the shock wave characteristics but are significantly in error for others. For example, the propagation velocity of the wave front decreases from its initial high value to essentially acoustic velocity within a distance of perhaps 20 charge radii away from the detonation. The shock wave energy spectrum, on the other hand, reflects the non-linearities of the initial high-amplitude wave out to ranges far beyond the point at which the wave velocity is acoustic.

1.3 The spectrum of the shock wave is, of course, a function of the charge weight. It is also a function of range, and for discrete frequency components of the broad band pulse the range dependency is different from the range dependency of a sinuoidal pulse of the same frequency. Because the peak pressure falls off with range at a greater rate than $1/R$ (the ideal acoustic wave decay) and the profile of the wave continues to broaden, the spectral distribution of the shock wave changes as the pulse propagates outward. These effects, which are not present in ideal acoustic waves, continue out to surprisingly great ranges.

*A resume' of this paper was presented at the Sixty-Fourth Meeting of the Acoustical Society of America, November 1962, Seattle, Washington.

**References are listed at the end of the report.

1.4 Since detailed data on shock waves several miles from small charges are sparse, it is frequently necessary to scale an available spectrum to other conditions of interest. It is clear from the above that if the spectrum is scaled as though the shock wave were propagating as an ideal acoustic wave, errors in scaled spectrum levels can result. The magnitudes of such errors depend upon the charge weights, the distances, and the frequencies involved. The relationships between explosion wave and ideal acoustic wave treatments of the shock wave energy spectrum are the subject of this paper.

1.5 In the so-called "explosion wave" treatment here the shock wave energy spectrum level as a function of charge weight and range is determined in the manner of Weston (ref. (5)). The shock wave is assumed to be a steep-fronted exponential wave, the peak pressure and decay constant of which are given by explosion similitude laws that describe the shock wave near the charge. The "ideal acoustic wave" treatment assumes that the pressure falls off with distance as $1/R$ and that there is no change in the pulse shape as the wave propagates. For both treatments shock wave energy spectrum functions are presented in the form of scaling relationships with which the spectrum for one set of experimental conditions (i.e., explosive composition, charge weight, range) can be scaled to a second set of experimental conditions.

1.6 Although only the acoustic wave treatment is denoted as "ideal", the shock wave treatment does, in fact, imply several idealizing assumptions. The range out to which similitude relationships obtain is open to question, since measurements of very low amplitude waves are highly subject to instrumental errors. In addition, the use of an exponential wave form implies the assumptions that:

(1) The rounding of the front due to dissipative effects (ref. (6)) does not decrease the spectrum level,

(2) The contribution of the slowly-decaying pressure in the tail of the shock wave is negligible, and

(3) The wave is propagating in an infinite, homogeneous medium so that the pulse shape, and hence the spectrum level, is not affected by either reflections (ref. (7)) or refraction (ref. (8)).

A comment concerning the range of variables over which these assumptions are reasonable is included.

2. SHOCK WAVE ENERGY SPECTRUM SCALING

2.1 For an exponentially decaying pulse the Fourier integral for the energy spectrum level is:

$$E(f) = \frac{2}{\rho c}\left[\frac{p^2\theta^2}{(1+\omega^2\theta^2)}\right] \tag{1}$$

where p is the initial pressure of the wave, θ is the decay constant, ρc is the acoustic impedance, and ω is the angular frequency, $\omega = 2\pi$ times the frequency in cps. For an explosion shock wave, the pressure and time constant in Equation (1) can be determined from similitude equations of the form:

$$p = k\,(W^{1/3}/R)^{\alpha} \tag{2}$$

$$\theta = m\,(W^{1/3}/R)^{\beta}.W^{1/3} \tag{3}$$

Here W is charge weight, R is distance from the detonation, and the coefficients and exponents are functions of the charge composition. Combination of Equations (1), (2), and (3) leads to the following general scaling factors that can be used to scale the shock wave energy frequency spectrum $E_1(f_1)$ for one set of conditions to the spectrum $E_2(f_2)$ for a second set of conditions:

$$E_2 = K_E E_1 \text{ when } f_2 = K_f f_1, \text{ where}$$

$$K_E = \left[\frac{k_2 m_2}{k_1 m_1}\right]^2 \left[\frac{W_2^{2(\alpha_2+\beta_2+1)/3}}{W_1^{2(\alpha_1+\beta_1+1)/3}}\right]\left[\frac{R_1^{2(\alpha_1+\beta_1)}}{R_2^{2(\alpha_2+\beta_2)}}\right] \tag{4}$$

$$K_f = \left[\frac{m_1}{m_2}\right]\left[\frac{W_2^{-(\beta_2+1)/3}}{W_1^{-(\beta_1+1)/3)}}\right]\left[\frac{R_1^{-\beta_1}}{R_2^{-\beta_2}}\right] \tag{5}$$

2.2 For a given explosive material the above spectrum scaling factors reduce to simple forms for two cases that are frequently of interest:

(a) Constant range, varying charge weight

$$K_E(W) = \left[\frac{W_2}{W_1}\right]^{2(\alpha + \beta + 1)/3}; \; K_f(W) = \left[\frac{W_2}{W_1}\right]^{-(\beta + 1)/3} \quad (6)$$

(b) Constant charge weight, varying range

$$K_E(R) = \left[\frac{R_1}{R_2}\right]^{2(\alpha + \beta)}; \; K_f(R) = \left[\frac{R_1}{R_2}\right]^{-\beta} \quad (7)$$

When both charge weight and range are varied, the products of these factors are used.

2.3 Since it has been found that the values of $\alpha = 1.13$ and $\beta = -0.22$ determined experimentally for TNT (ref. (4)) are typical of the exponents for most high explosive materials*, these values are used here to compare explosion wave and ideal acoustic wave scaling factors. For an ideal acoustic wave there is no spreading of the profile (i.e., $\beta = 0$) and the pressure decays as $1/R$ (i.e., $\alpha = 1.0$). In Table I the spectrum scaling factors of Equations (6) and (7) for TNT and for an ideal acoustic wave are summarized.

TABLE 1. SPECTRUM SCALING FACTORS

	Constant Range, Varying Weight	Constant Weight, Varying Range
TNT ($\alpha = 1.13, \beta = -0.22$)	$K_E = (\frac{W_2}{W_1})^{1.27}$; $K_f = (\frac{W_2}{W_1})^{-0.26}$	$K_E = (\frac{R_1}{R_2})^{1.82}$; $K_f = (\frac{R_1}{R_2})^{0.22}$
Ideal Acoustic ($\alpha = 1.0, \beta = 0$)	$K_E = (\frac{W_2}{W_1})^{1.33}$; $K_f = (\frac{W_2}{W_1})^{-0.33}$	$K_E = (\frac{R_1}{R_2})^{2.0}$; $K_f = 1.0$

The ideal acoustic factors of Table 1 correspond to the spectrum scaling constants given by Weston in reference 5. In figures 1 and 2 the functions of Table 1 are shown graphically. Dashed lines in these figures represent the TNT scaling factors and solid lines the ideal acoustic wave factors. For convenience, the values of K_E are shown in decibels ($10 \log K_E$) so that they

*The effect of charge composition on spectrum level is discussed in Appendix 2.

can be simply added to or subtracted from a known spectrum level that is in dB.

2.4. In figure 1 the scaling factors of Equation (6), for the case of a constant range and varying charge weight, are shown. Here the curves are normalized to a charge weight of one pound, so that if the spectrum for a one-pound charge is known at some particular range, the spectrum for other charge weights at that same range can be determined from scale factors read directly from figure 1. For example, using explosion factors the spectrum ($E_2(f_2)$) of a 10-pound charge (at the same range) is obtained from the spectrum of a one-pound charge ($E_1(f_1)$) by adding 12.7 dB to the energy levels and multiplying the frequencies by a factor of 0.55. In other words, if, for a one-pound charge, $E_1 = y$ (dB) at $f_1 = x$ (cps), at that same range, for a 10-pound charge

$$E_2 = y \text{ (dB)} + 12.7 \text{ at } f_2 = 0.55\, x \text{ (cps)}.$$

Similarly, if the one-pound spectrum is being scaled to that of a charge weighing only 0.01 pounds, the explosion scaling factors would be:

$$E_2 = y \text{ (dB)} - 25.5 \text{ dB at } f_2 = 3.3\, x \text{ (cps)}$$

Thus, at any given range, increasing the charge weight increases the spectrum level and shifts the spectrum towards lower frequencies, while decreasing the charge weight lowers the level and shifts the spectrum to higher frequencies. As can be seen from the two sets of curves in figure 1, the differences between explosion and ideal acoustic scaling of the spectrum arise from the frequency scaling factor, rather than the level scaling factor*. It is also apparent from figure 1 that the two sets of scaling factors give essentially the same results for charge weights between about 0.1 and 100 pounds. For charge weights less than 0.1 pounds, however, significant errors in the spectrum could be introduced by use of the ideal acoustic factors; the magnitude of the error would depend upon the slope of the level at the particular frequencies of interest.

2.5 In figure 2 the scaling factors of Equation (7) for the case of constant charge weight and varying range are shown. Here the values are normalized to a 100-yard range; scaling factors are found as in the preceding example. Figure 2 shows that for a given charge weight the spectrum level is decreased and shifted

*The greater significance of the frequency factor arises, of course, from the use of decibels, rather than ratios, to express spectrum levels. Numerically the two factors are about equally significant.

towards lower frequencies as range is increased. Thus, although the level is lowered by either a decrease in weight or an increase in range, the frequency shifts are in the opposite directions for the two cases. As was the case for constant range, the difference in explosion and ideal acoustic scaling for spectra from different charge weights is due to the frequency shift rather than the level change. From figure 2 it is also apparent that there are only relatively minor differences in the two sets of scaling factors for ranges between about 10 and 1000 yards. However, in scaling the spectrum at 100 yards back to that at 1 yard, the reference range customarily used for sonar, differences between the two methods of scaling may be significant*. As noted previously, the magnitude of such scaling errors varies with frequency.

2.6 A final example of the use of figures 1 and 2 is given for the case of changing charge weight and range simultaneously. The spectrum for a one-pound charge at 100 yards can be scaled to that of a 0.1-pound charge at 10 yards with the following factors:

10 log K_E due to charge weight decrease	is	-12.7 dB(Fig.1)
10 log K_E due to range decrease	is	+18.2 dB(Fig.2)
so that the level is increased by		+ 5.5 dB

K_f due to charge weight decrease is 1.82 (Fig. 1)

K_f due to range decrease is 1.65 (Fig. 2)

so that frequencies are multiplied by (1.82 x 1.65 = 3.0)

2.7 The curves of figures 1 and 2 are directly applicable to weight and range <u>ratios</u>, as well as to the values of weights and ranges shown. In other words, the spectrum of a 5.76-pound charge can be scaled to the spectrum of a 0.0576-pound charge with factors read at 10^{-2} on the abscissa of figure 1. Similarly, factors for scaling a spectrum from 50 yards to 5000 yards can be read at 10^4 (since the curves shown are normalized to 10^2) on the abscissa of figure 2. It should also be noted that the curves of figures 1 and 2 can be used to scale spectra between any two values of weight and range, even though the

*Weston (Ref.(5)) considers 100 yards to be a suitable reference range for determining the acoustic source level of an explosive charge, since finite amplitude effects are comparatively small beyond about 100 yards; this distance is also typical for the range from charge to first boundary reflection. There is, however, a tendency to scale Weston's 100-yard data back to a 1-yard range, e.g., reference 9.

appropriate factors cannot be read directly from the ordinate scales shown. For such use the scaling values that are needed are those which would shift the correction factor curves shown here so that 10 log $K_E = 0$ and $K_f = 1.0$ for the weight and range of the known spectrum. For example, the factors for scaling the spectrum at one yard from a given charge weight to the spectrum for that same charge at 1000 yards can be obtained from the values shown for these two ranges in figure 2. In this case, using explosion factors, spectrum levels would be decreased by 55 dB (the difference between the +37 dB value at 1 yard and the -18 dB value at 1000 yards) and the corresponding frequencies would be multiplied by(0.60/2.75 = 0.22), the ratio of the factors at the two ranges.

3. COMPARISON WITH EXPERIMENTAL RESULTS

3.1 In figure 3 the energy flux spectrum levels at one yard and at 100 yards from a one-pound TNT charge are shown. The uppermost, solid curve was obtained by digital analysis of a shock wave recorded 2.5 ft from the charge and scaled to a one-yard range using Equation (7). Experimental conditions and details of the spectral analysis for this curve and the measured spectrum of figure 4 are given in Appendix 1. The 100-yard data (open circles) of figure 3 are the octave band measurements given by Weston in Table 2 of reference 5*. The two dashed curves for the spectrum at 100 yards were scaled from the 1-yard spectrum with the factors of figure 2; the long dash represents the explosion factors and the short dash represents ideal acoustic factors.

3.2 The measured and scaled spectra at a range of 280 yards from a 1.8-pound charge are shown in figure 4. The solid curve is the spectrum computed from a measured shock wave and the dashed curves were scaled from the 1-pound, 1-yard curve of figure 3. Again, the long dash indicates scaling with explosion factors and the short dash indicates scaling with ideal acoustic factors. Since both charge weight and range were changed here, both figures 1 and 2 were used for the scaled curves of figure 4.

3.3 At both 100 yards and 280 yards the explosion wave scaling factors give excellent agreement between measured and scaled shock wave spectra; except at the highest frequency for the 100-yard data (fig. 3) the scaled results are within 1 dB of the measured values. Scaling with ideal acoustic factors gives

*In the data of reference 5, contributions from the bubble pulse were predominant for frequencies below about 300 cps, as is indicated on figure 3. Bubble pulses were not considered in the digital computations represented by solid lines in figures 3 and 4.

spectra that are too high at the higher frequencies. As would be expected from figures 1 and 2, the errors are greater for the larger charge and longer range of figure 4 than for the scaled conditions of figure 3. In figure 4, although the values scaled with ideal acoustic values are only about 2 dB too high at 1 kc, the error increases to about +10 dB at a frequency of 10 kc.

4. ALLOWABLE RANGE OF VARIABLES

4.1 The explosion wave scaling factors described here are applicable only so long as the shock wave exhibits the non-linear characteristics on which the scaling laws are based. The ranges and weights to which this restricts these curves are the ranges and weights for which the explosion similitude equations describe the range dependence of the pressure and time constant of the shock wave. Although the similitude functions are not applicable right at the charge surface, experiments have shown that by the time the shock wave has progressed outward to only about 7 charge radii, the peak pressure and time constant begin to follow the similitude relationships (ref. (1)). The real question is how far out from the charge these same relationships are valid. Arons (ref. (4)) found that the $1/R^{1.13}$ pressure decay and the time constant function used here could be used to describe the data down to values of $W^{1/3}/R$ (cube root of weight in pounds and range in yards) of about 3.33×10^{-4}. Only this one set of data is available for such small values of $W^{1/3}/R$ and the experimental scatter is great. Furthermore, it is clear that at some low value of $W^{1/3}/R$ the exponents of the similitude equations must approach the acoustic values; shock wave theory predicts that they would have done so at shorter ranges than experiment indicated. In view of these considerations it is recommended that the explosion scaling laws developed here be restricted to the interval $4 \times 10^{-4} \le W^{1/3}/R \le 3$ where W is charge weight in pounds, R is range in yards.

4.2 In addition to the restriction on charge weights and ranges noted above, these scaling laws also carry a frequency restriction. The scaling functions developed here assume a steep fronted (i.e., zero time of rise to the maximum pressure) exponential wave and may overestimate the spectrum level at high frequencies. Although the shock wave is steep-fronted near the charge, as the wave propagates outward the front becomes increasingly rounded due to dissipation. At the same time, there is a finite amplitude effect which tends to sharpen up the front of the propagating wave. Arons has considered the combined results of these effects and gives theoretical values of rise time vs range for various charge sizes in figure 13 of reference (6). The influence of the rise time on the shock wave spectrum is shown for several hypothetical curves in reference (7).

From references (6) and (7) it appears that, for the stated interval of $W^{1/3}/R$, spectra obtained with the functions shown here are probably not seriously in error for frequencies below about 20 kc.

4.3 The lower limit of frequencies for which these scaling laws are suitable is also dependent upon the exponential approximation, which neglects the slowly-decaying pressure of the shock wave tail. To compensate in part for the resulting loss of low-frequency components, Weston (ref. (5)) used an average value of time constant (θ = impulse/peak pressure) for computing his theoretical spectrum of a one-pound charge at 100 yards. Such treatment would not be feasible for the scaling methods shown here, since the correction to the similitude θ that would be required would vary depending upon the charge weights and ranges of interest. Furthermore, for charges fired at fairly shallow depths, the low-frequency portion of the spectrum is subject to considerable variation due to bubble pulses and surface reflections, which are functions of charge and receiver depths. Consequently, for general use it seems more appropriate to use the similitude values of time constant with the exponential approximation, and restrict the frequency range over which the functions are employed. For small charges the spectrum computed for the exponential approximation agrees with the measured spectrum within 2 dB or better for frequencies above about 500 cps (see Appendix 1). Since the entire pressure time curve, as well as the peak pressure, can be scaled in terms of $W^{1/3}/R$, one would expect a similar relationship for other weights and ranges. Consequently, 500 cps is taken as a reasonable lower frequency limit for the scaling relationships given here.

4.4 To summarize the above restrictions, the scaling laws given here should not be used outside the following ranges of variables:

$$4 \times 10^{-4} \leq W^{1/3}/R \ (\text{lb}^{1/3}/\text{yd}) \leq 3$$

$$0.5 < \text{frequency (kc)} < 20$$

Some small errors due to the assumptions used may be present at the extreme values of these ranges.

5. SUMMARY

5.1 Scaling factors with which the underwater shock wave energy spectrum for one set of experimental conditions (i.e., explosive composition, charge weight, and range) can be scaled to another set of conditions are derived by combining shock wave similitude equations with the Fourier integral for the spectrum

of an exponential pulse. These factors are given by Equations (4) and (5).

5.2 Scaling factors are shown graphically for the case of constant range with varying charge weight (fig. 1) and the case of constant charge weight with varying range (fig. 2). Examples of the use of these graphs are included.

5.3 For comparison with shock wave scaling factors, analogous scaling factors for an ideal acoustic wave (i.e., one in which the pressure decay with distance is $1/R$ and the wave shape is constant) are also shown in figures 1 and 2. For charge weights between about one and 100 pounds, and for ranges between about 10 and 1000 yards, there is little difference between these two sets of scaling factors. If shorter ranges and smaller charge weights are involved, however, significant errors can result if shock wave spectra are scaled with the ideal acoustic wave factors.

5.4 Spectra obtained from digital analysis of shock waves measured at one yard from a one-pound charge and at 280 yards from a 1.8-pound charge are shown in figures 3 and 4. Data given in reference (5) for octave band measurements at 100-yards from a one-pound charge are also shown in figure 3. The measured one-pound,one-yard spectrum is scaled to the other two sets of conditions with both the explosion factors and the ideal acoustic factors of figures 1 and 2.

5.5 Spectrum scaling with explosion wave factors gives better agreement with the data. Values scaled with ideal acoustic factors are too high at higher frequencies, with worse agreement for the longer range and larger weight case (fig. 4). In scaling from the one-pound, one-yard data to 1.8-pound, 280-yard levels, the error with ideal acoustic scaling is only about + 2 dB for frequencies below 1 kc,but increases to + 10 dB at a frequency of 10 kc.

5.6 The allowable range of variables for the spectrum scaling functions given here is estimated to be:

$$0.5 < \text{frequency (kc)} < 20$$

$4 \times 10^{-4} \leq W^{1/3}/R \leq 3$, where W is charge weight in pounds and R is range in yards.

5.7 Spectral analysis of the measured shock waves is discussed in Appendix 1.

5.8 A method for estimating the effect of charge composition on spectrum level is given in Appendix 2.

REFERENCES

(1) R. H. Cole, Underwater Explosions, Princeton University Press, 1948.

(2) M. Ewing and J. L. Worzel, "Long-Range Sound Transmission", Geological Society Memoir 27, 1948.

(3) "Physics of Sound in the Sea", Div. 6, Vol. 8 of NDRC Summary Technical Reports.

(4) A. B. Arons, "Underwater Explosion Shock Wave Parameters at Large Distances from the Charge", J. Acous. Soc. Am., 26, 343 (1954).

(5) D. E. Weston, "Underwater Explosions as Acoustic Sources", Proc. Phys. Soc. (London) B76, 233 (1960).

(6) A. B. Arons, D. R. Yennie, and T. P. Cotter, Jr., "Long-Range Shock Propagation in Underwater Explosion Phenomena II", NAVORD Report No. 478, October 1949.

(7) T. F. Johnston and R. W. Raitt, "Transmission of Explosive Impulses in the Sea", Univ. of California Div. of War Research Report, UCDWR No. U8 Dec. 1942.

(8) R. R. Brockhurst, J. G. Bruce, Jr., and A. B. Arons, "Refraction of Underwater Explosion Shock Waves by a Strong Velocity Gradient", J. Acous. Soc. Am., 33, 452 (1961).

(9) R. W. Hasse, Jr., "A Comparison of Propagation Loss Measurements Using Explosive and Sinusoidal Acoustic Signals", USL Research Report No. 498, Dec. 1960.

(10) J. P. Slifko, "Underwater Shock Wave Frequency Spectrum Analysis 1. Simple Wave Forms Approximating Measured Pressure Pulses in Water", NOLTR 61-94, July 1961.

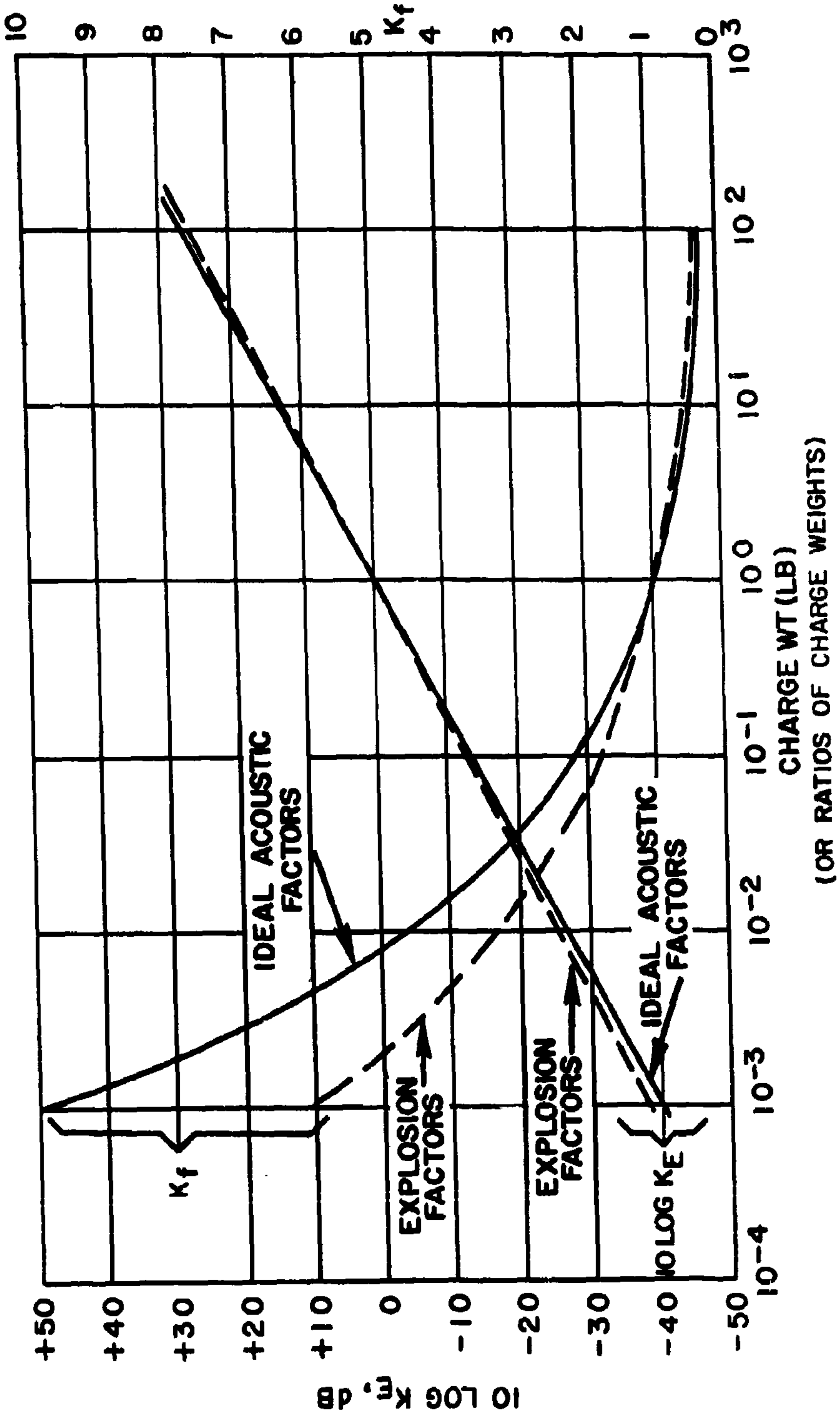

FIG. 1 FACTORS FOR SCALING ENERGY SPECTRUM FOR 1-LB TNT CHARGE TO OTHER CHARGE WEIGHTS, SAME RANGE

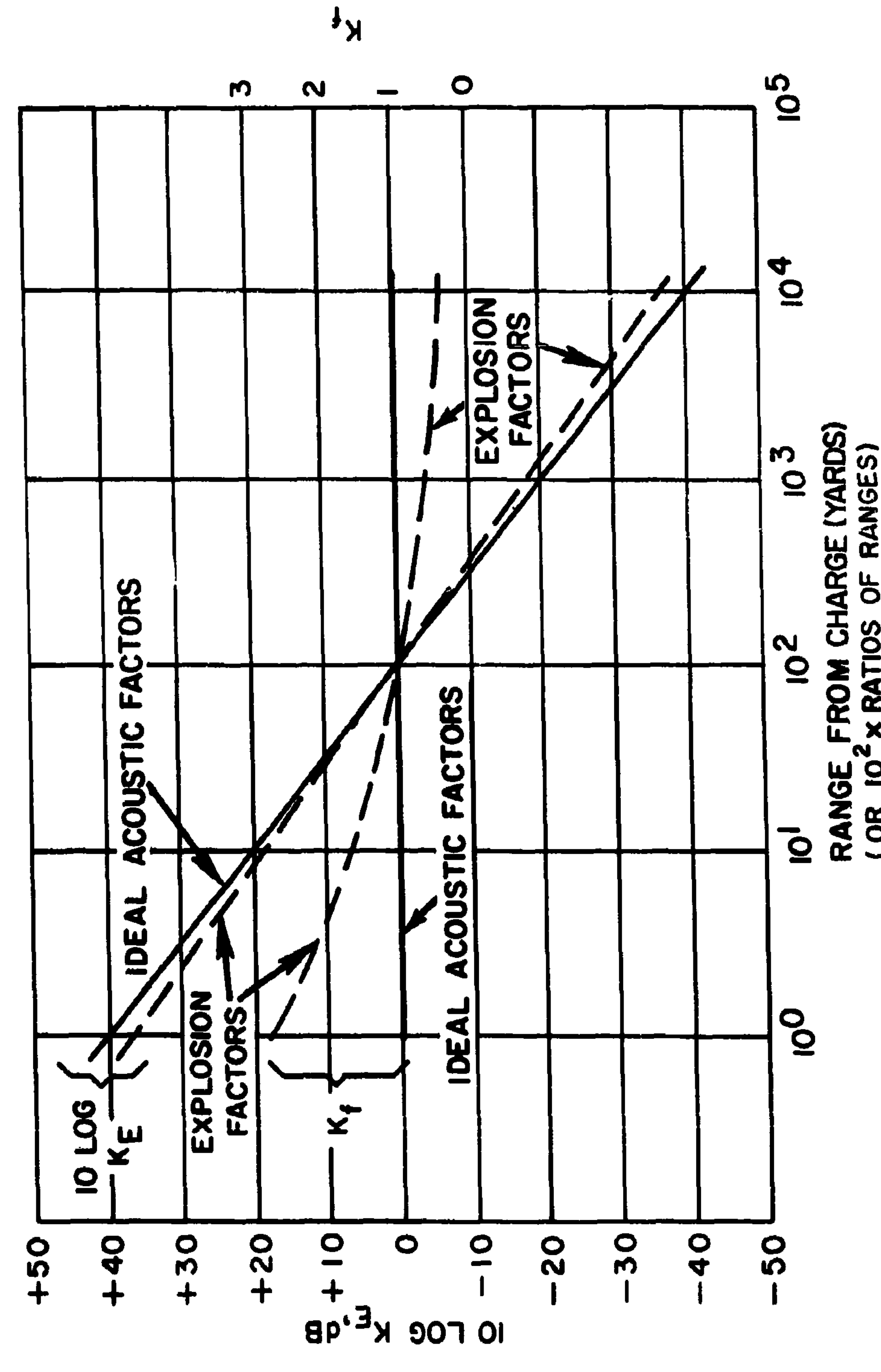

FIG. 2 FACTORS FOR SCALING ENERGY SPECTRUM OF GIVEN WEIGHT TNT CHARGE FROM 100 YARD TO OTHER RANGES

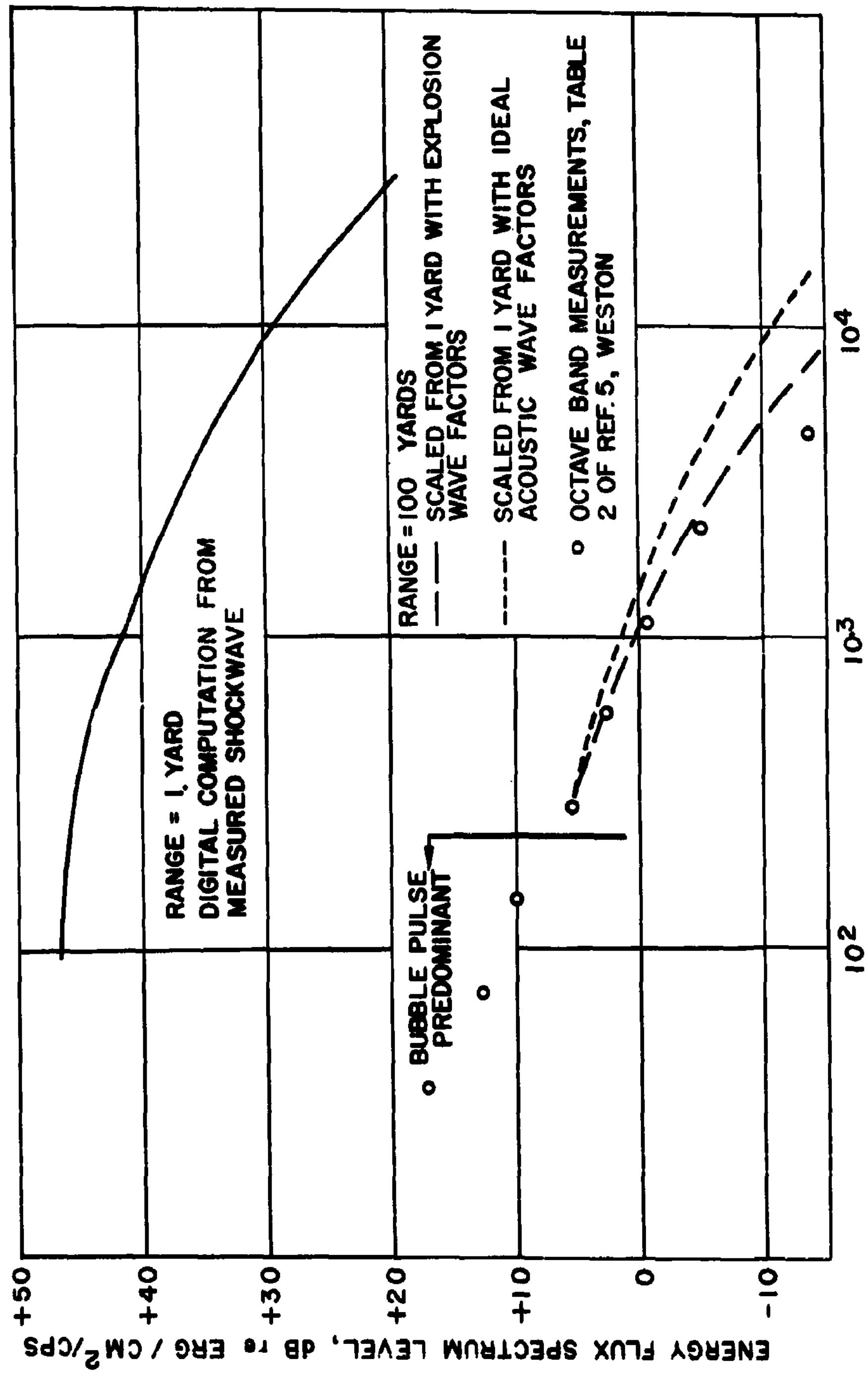

FIG. 3 ENERGY FLUX SPECTRA FOR 1 LB TNT CHARGES AT 1 YARD AND 100 YARDS

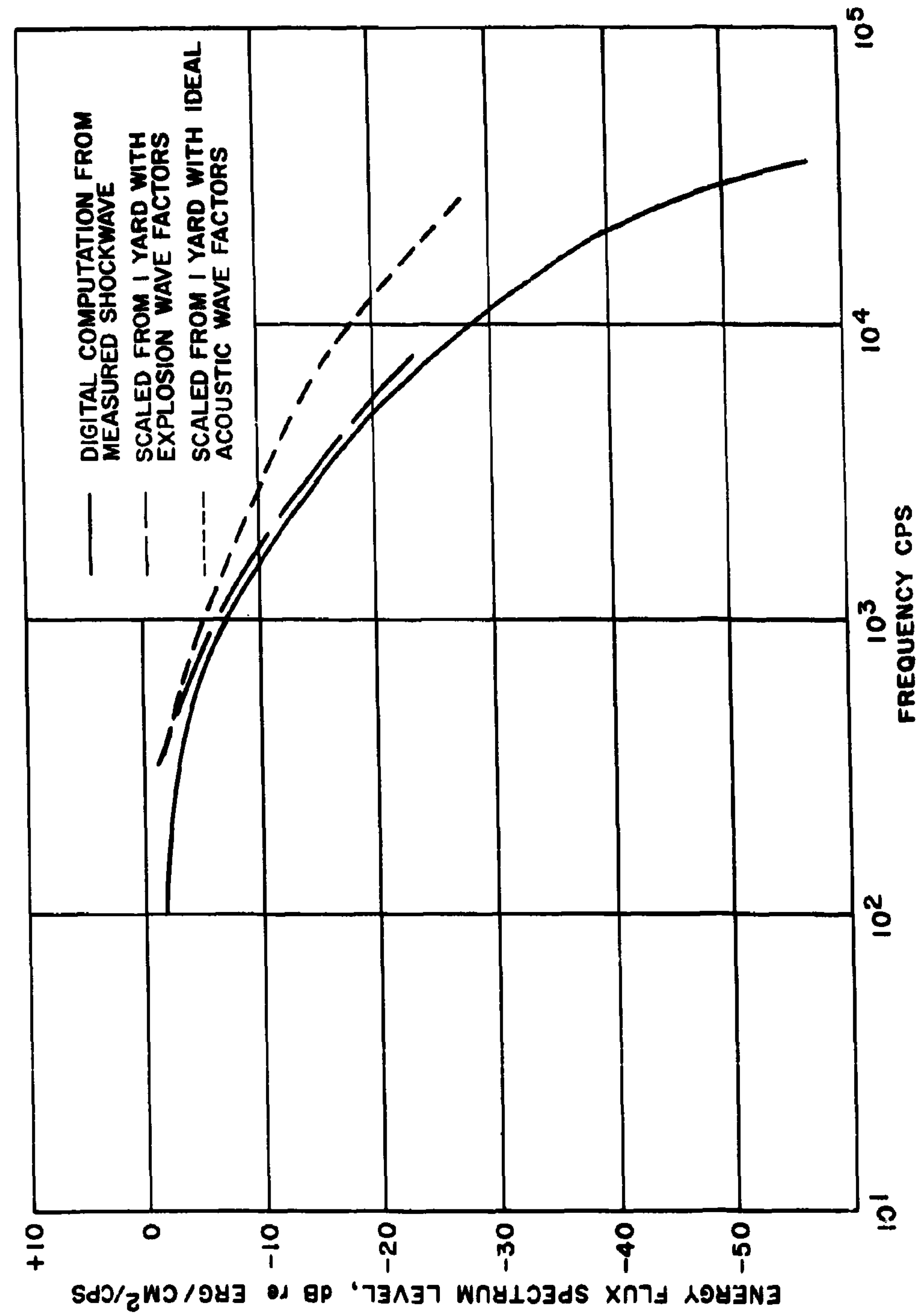

FIG. 4 MEASURED AND SCALED ENERGY FLUX SPECTRA AT 280 YARDS FROM A 1.8 POUND TNT CHARGE

APPENDIX 1. SPECTRAL ANALYSIS OF MEASURED SHOCK WAVES

1. The energy spectra of several underwater pressure-time curves recorded at various distances from small point charges were computed by Fourier analysis, using an IBM 7090 machine program developed for use with arbitrary wave forms. The functions evaluated were:

$$A(f) = \frac{\tau}{N}\sqrt{\left|\sum_{n=1}^{N+1} p\ (T)\ \sin \omega T\right|^2 + \left|\sum_{n=1}^{N+1} p\ (T)\ \cos \omega T\right|^2}$$

$$E(f) = \frac{2}{\rho c}\left|A(f)\right|^2$$

where $T = (n - \frac{1}{2})\frac{\tau}{N}$ and $\omega = 2\pi f$

τ is the total time of integration (seconds); N is the number of equal time increments in the interval τ; ω is the angular frequency (radians) and p(T) is the amplitude (dynes/cm^2) of the pressure-time curve at the mid-point of the nth time increment.

2. In the energy calculations an average value of

$$\rho c = 15.25 \times 10^4 \text{ grams/cm}^2\text{/sec}$$

was used. Variations of water temperature between -2°C and 28°C, salinities between 34°/00 and 36°/00, and densities between 1.0 g/cm^3 and 1.05 g/cm^3 result in a spread of only 1.74 x 10^4 g/cm^2/sec in the values of ρc.

3. Spectra obtained from such point-wise integration, which takes into account the slowly-decaying tail of the shock wave, differ from theoretical spectra for the initial exponential portion of the pulse only at frequencies below about 1000 cps for the conditions measured here. At a frequency of about 500 cps the spectrum for the exponential approximation is down 2 dB from the measured spectrum; the difference increases as frequency decreases.

4. The spectrum at 1 yard from a 1-pound charge (fig. 3 of report) was computed from an oscillograph recording made about one yard away from a one-pound charge in the Potomac River*; both charge and tourmaline gage were suspended at mid-depth in

*Measurements were actually made at 2.5 feet from the charge and the spectrum was then scaled to a one-yard range using Equation (7).

about 30 feet of water. The record for a 1.8-pound charge (fig. 4 of report) was obtained from an oscillograph playback of a tape recording made at sea. In this case the charge was fired at a depth of 356 feet and recorded on a hydrophone 296 feet deep and 844 feet (slant range) from the charge. The frequency response of the oscilloscope recording equipment was good from essentially zero to 100 kc; with the tape recording equipment the frequency range was only 50 cps to 20 kc. The measured spectrum for the latter record (fig. 4) reflects the poor low-frequency response of the system.

5. For both of these shock wave records integration was carried out over the time interval between the initial pressure discontinuity at the shock wave arrival and the subsequent decay of the wave to approximately zero pressure. The integration intervals actually used were 1.25 millisecs for the one-pound charge and 1.29 millisecs for the 1.8-pound charge. Slifko (ref. 10) has shown that for an exponential wave the interval of integration has little influence on the spectrum, provided the interval is at least as long as five times the decay constant of the wave. Here the integrations were extended over about 10 and 4 times the decay constants of the exponential portions of the pulse for the 1-pound and 1.8-pound charges, respectively.

ACKNOWLEDGEMENT

The author is indebted to Mr. J. P. Slifko and Mr. W. H. Faux for the spectral analyses of measured shock waves.

APPENDIX 2

ESTIMATING THE EFFECT OF CHARGE COMPOSITION ON SPECTRUM LEVEL FOR CONSTANT CHARGE WEIGHT AND RANGE

1. Although TNT, or some explosive material that is very similar in performance and propagation characteristics, is usually used for underwater explosive sound sources, the effect of explosive composition is of interest when new explosive materials are considered for such applications. When the similitude constants for any explosive material are known the spectrum scaling factors, such as those shown for TNT in figures 1 and 2 of the report, can be determined from Equations (4) and (5) of the report. In many cases, however, one is primarily interested in an estimate of the approximate difference between the spectrum of the new material and that of a known material, say TNT, for the same charge weight and range. For such comparisons the following equations relating ϵ, the change in shock wave energy spectrum level (in dB), to the similitude parameters (Equations (2) and (3) of report) for the two materials are useful:

$$\epsilon = 10 \log E_2(f) - 10 \log E_1(f) = C + X_E + X_f \qquad (A.1)$$

where $$C = 20 \log (k_2 m_2)/(k_1 m_1) \qquad (A.2)$$

$$X_E = 20 (\alpha_2 - \alpha_1 + \beta_2 - \beta_1) \log (W^{1/3}/R) \qquad (A.3)$$

$$X_f = 10 \log \left[(1 + \omega^2 \theta_1^{\,2})/(1 + \omega^2 \theta_2^{\,2})\right] \qquad (A.4)$$

and $W_1 = W_2$, $R_1 = R_2$.

The quantity C is indicative of the relative energies of the two materials and is independent of charge weight and range. The variables X_E and X_f are dependent upon the similitude, or propagation, characteristics of the shock waves of the two materials; the charge weight and range for which the two materials are being compared enter directly into the X_E factor, and in the θ terms of the X_f factor.

2. An approximate comparison of the two materials can usually be made very easily from these equations. In practice it has been found that the sum of the similitude exponents $(\alpha_2 - \alpha_1 + \beta_2 - \beta_1)$ in Equation (A.3) is very nearly zero for most high explosive materials, so that X_E is negligibly small. In

addition, for charge weights of no more than a few pounds, X_f for extreme values of frequency can be approximated by:

$X_f = 0$ for frequencies below about 1 kc

$X_f = 20 \log (\theta_1/\theta_2)$ for frequencies above about 8 kc

so that the effects of this term on spectrum level can be estimated quickly for several values of $W^{1/3}/R$.

3. For the occasional case in which more detailed comparisons are needed, graphs of the functions X_E and X_f are convenient. Two such graphs, covering the range of variables customarily encountered with high explosive underwater charges, are shown in figures A.1 and A.2. In figure A.1 the quantity X_E is plotted vs $W^{1/3}/R$ with

$$\eta = (\alpha_2 - \alpha_1 + \beta_2 - \beta_1) \tag{A.5}$$

as parameter. Note that in this graph the range (R) is used in feet, as is the custom for explosion similitude curves, rather than in yards, as in the body of the report. In figure A.2, X_f vs $\omega\, \theta_1$ is shown for several values of θ_2/θ_1. The use of Equation (A.1) and figures A.1 and A.2 for comparing the energy spectrum for some hypothetical explosive "Y", which is uncommonly different from TNT, with that of TNT is illustrated below.

4. Assume the explosive "Y", for which the subscript "2" is used, has the following similitude values:

$$p_2 = 1.10 \times 10^4 \, (W^{1/3}/R)^{1.05}, \text{ peak pressure in psi} \tag{A.6}$$

$$\theta_2 = 0.10 \, (W^{1/3}/R)^{-0.09} \, W^{1/3}, \text{ time constant in millisec} \tag{A.7}$$

Here W is in pounds and R in feet. From reference (4) the corresponding equations for TNT, which has the subscript "1", are:

$$p_1 = 2.16 \times 10^4 \, (W^{1/3}/R)^{1.13} \tag{A.8}$$

$$\theta_1 = 0.058 \, (W^{1/3}/R)^{-0.22} \, W^{1/3} \tag{A.9}$$

The comparison will be made for equal ranges and weights at two values of $W^{1/3}/R$, viz., (1) $W^{1/3}/R = 1.0$, which is quite near the charge and might be thought of as an approximate comparison of source levels, and (2) $W^{1/3}/R = 10^{-3}$, which corresponds to 1000 feet from a one-pound charge and indicates the differences in propagation of the two shock waves. The spectrum levels will be compared at two frequencies, 1 and 8 kc.

From Equation (A.2),

$$C = 20 \log (k_2m_2)/(k_1m_1) = 20 \log \frac{1.10 \times 0.10}{2.16 \times 0.058} = -1.1 \text{ dB}$$

From Equation (A.5), $\eta = 1.05 - 1.13 - 0.09 + 0.22 = + 0.05$; and from figure A.1, $X_E = 0$ dB at $W^{1/3}/R = 1.0$ and $X_E = -3$ dB at $W^{1/3}/R = 10^{-3}$.

For computing the quantity X_f the actual charge weight must be used, since it appears in the equation for θ. If a charge weight of one pound is assumed then:

@ $W^{1/3}/R = 1.0$, $\theta_2 = 10^{-4}$ sec and $\theta_1 = 0.58 \times 10^{-4}$ sec;

$\theta_2/\theta_1 = 1.725$

@ $W^{1/3}/R = 10^{-3}$, $\theta_2 = 1.86 \times 10^{-4}$ sec and $\theta_1 = 2.65 \times 10^{-4}$ sec;

$\theta_2/\theta_1 = 0.702$

and with figure A.2 the X_f values tabulated below can be read.

		$W^{1/3}/R = 1.0$		$W^{1/3}/R = 10^{-3}$	
f(kc)	ω	$\omega\theta_1$	X_f(dB)	$\omega\theta_1$	X_f(dB)
1	0.63×10^4	0.365	-1	1.67	+2
8	5.03×10^4	2.92	-4	13.3	+3

Summarizing the above factors (Equation A.1) the values for ϵ in dB are:

	$W^{1/3}/R = 1.0$	$W^{1/3}/R = 10^{-3}$
f = 1 kc	ϵ = -2 dB	ϵ = -2 dB
f = 8 kc	ϵ = -5 dB	ϵ = -1 dB

Thus, very near the charge the energy spectrum level of a one-pound charge of explosive "Y" is below that of TNT, the difference increasing from -2 dB at 1 kc to -5 dB at 8 kc. Because of differences in the propagation laws for the two materials, however, at 1000 feet from the burst the relationship of the two spectra has changed; here the level for explosive "Y" is still 2 dB below that of TNT at the lower frequency (1 kc) but only 1 dB below the TNT values at 8 kc.

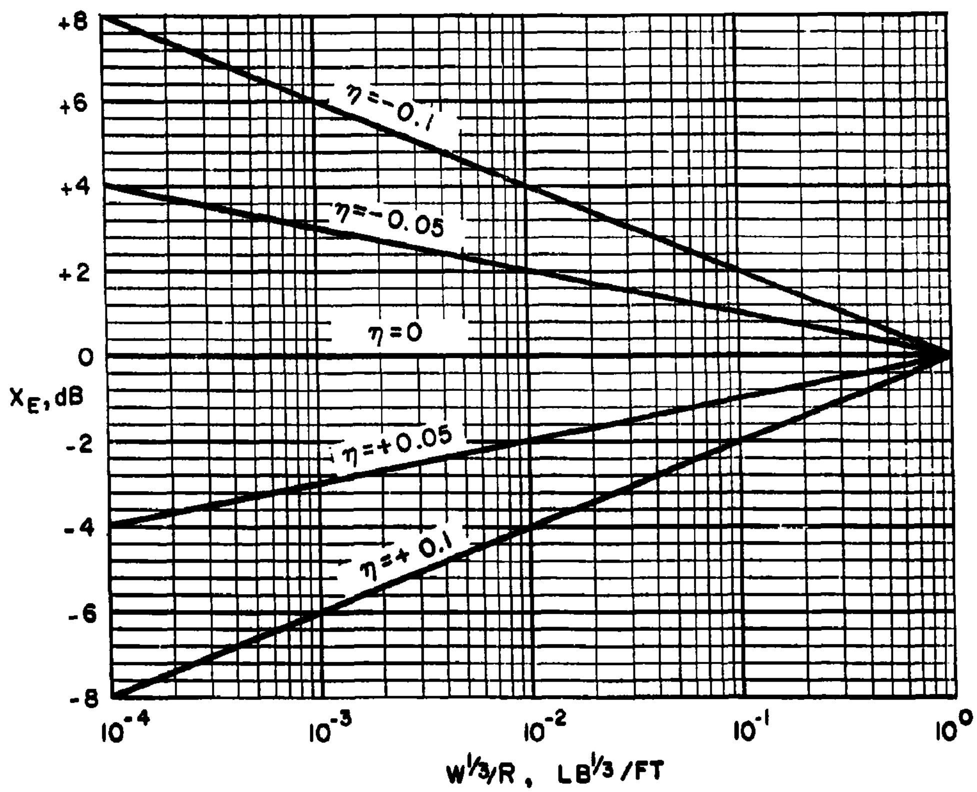

FIG. A.1 X_E AS FUNCTION OF $W^{1/3}/R$ WITH η AS PARAMETER

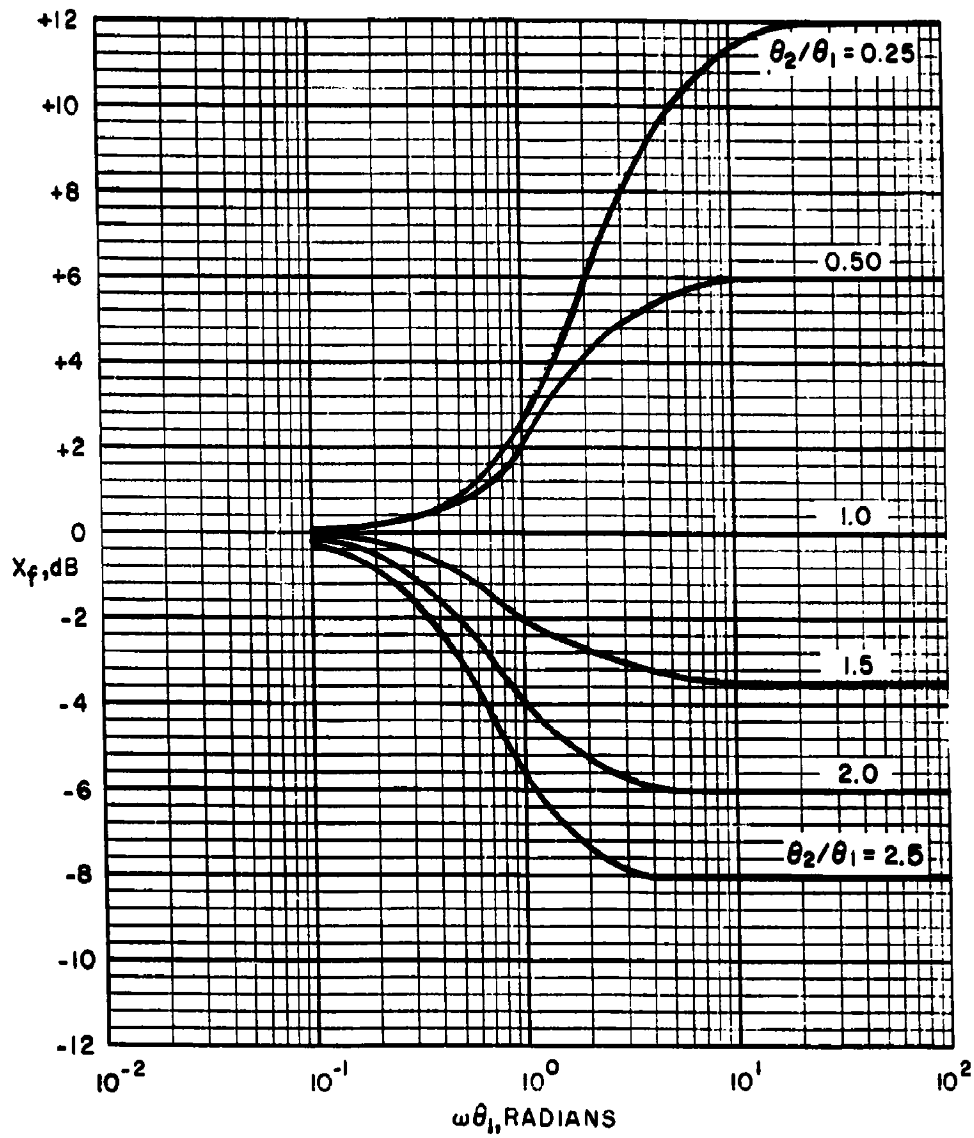

FIG. A.2 X_f AS FUNCTION OF $\omega\theta_1$ WITH (θ_2/θ_1) AS PARAMETER

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UNCLASSIFIED

Scaling factors with which the energy spectrum for an underwater shock wave can be scaled from one set of conditions (i.e., explosive composition, charge weight, and range) to a second set of conditions are derived. Analogous scaling factors for an ideal acoustic wave are also shown. In both cases the explosion scaling is in agreement with the data whereas the ideal acoustic scaling gives spectrum levels that are too high at the higher frequencies.

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